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Heat Transfer Analysis of a Stretching Sheet in TRI Particle-Enhanced Nanofluid Systems

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Abstract: This research investigates the cooling efficiency of tri-particle nanofluids, specifically focusing on a combination of water as the base fluid, with titanium dioxide, cobalt ferrite, and magnesium oxide nanoparticles. The study targets the application of these nanofluids on industrial stretching sheets, where the cooling rate directly influences the final quality of the sheet. The aim is to identify a more effective coolant, enhancing heat exchangers by utilizing a combination of nanoparticles. The research models the flow of tri-hybrid nanofluids over a stretching sheet, considering both liquid and dust phases. Key parameters such as velocity, temperature, skin friction coefficient, and Nusselt number are analyzed, with results presented in visual and tabular formats. The study found that the velocity and heat gradient of the fluid and dust phases increase in opposite directions. Continuity equations, adjusted for density, viscosity, and temperature variations, are transformed into ordinary differential equations (ODEs), which are solved using mathematical software. The findings demonstrate that tri-hybrid nanofluids significantly improve heat transfer, suggesting their potential as superior coolants in various industrial applications, offering flexibility in nanoparticle combinations to optimize performance.

Keywords: Nano Fluids and Nano-particles; Stretching Sheet; Ternary Nanofluids; Hybrid Nanofluids; Ethylene and Diethylene Glycol; Refrigerants and Nanoscience; Angiography and Magnetohydrodynamics (MHD).

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1. Introduction

Traditional heat transfer models often struggle to accurately predict thermal behaviors, particularly in scenarios involving micro-scale dimensions or rapid temperature fluctuations. These limitations are addressed by the modification rule, which introduces enhanced accuracy in modeling heat transfer processes. The Cattaneo-Christov (CC) formulation, a significant advancement over the classical Fourier law, plays a pivotal role in this context. Unlike the Fourier model, which assumes an instantaneous propagation of thermal signals, the CC model incorporates thermal relaxation time, making it more suitable for analyzing heat transfer in materials with micro-scale dimensions or in cases involving rapid thermal transients. The CC formulation has proven versatile, finding applications across various fields such as materials science, nanotechnology, and aeronautical engineering. By integrating the CC model, researchers can predict heat transfer characteristics with greater

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precision, accounting for variables that were previously overlooked. This enhanced predictive capability is crucial for understanding thermal processes, potentially leading to the development of new materials and technologies that can transfer heat more efficiently [19].

A prominent application of the CC model can be seen in the work of Khan and Pop [25], who investigated the two-dimensional (2D) unstable Maxwell fluid flow over a stretched cylinder. Their research revealed that lower levels of the thermal relaxation parameter result in reduced heat conduction. This finding underscores the importance of the CC model in scenarios where precise control over thermal conductivity is required, such as in the design of advanced heat exchangers or cooling systems in microelectronic devices. Sakiadis [6] further extended the application of the CC theory by studying the processes of mass and heat transport in a time-dependent Maxwell fluid flow. His research highlighted that with an increase in Maxwell and magnetic parameters, there is a corresponding decrease in the velocity profile. This observation is significant in the context of magnetic nanofluids, where controlling the velocity and heat transfer rate is essential for optimizing performance in applications like targeted drug delivery or magnetic refrigeration [21].

Elbashbeshy and Aldawody [7] expanded upon this by exploring the non-linear radiative magnetohydrodynamic (MHD) Maxwell nanofluid flow through a stretched cylinder in a Darcy-Forchheimer porous medium. Their study found that an increase in the Maxwell parameter value leads to a decrease in the velocity profile. This behavior is particularly relevant in porous media applications, where fluid flow and heat transfer are influenced by both the porosity of the medium and the magnetic properties of the fluid. The work of Gaffar [8] on non-linear convective Maxwell fluid flow across a stretched sheet delves into the impact of heat source/sink interactions as well as variable thermal conductivity. His findings indicate that as the elastic parameter increases, so does the skin friction coefficient. This relationship between elasticity and friction is crucial for understanding the behavior of viscoelastic fluids in industrial processes, such as polymer extrusion or the manufacturing of synthetic fibers, where precise control over material properties is required [26].

Hemmat et al. [9] contributed to this body of knowledge by investigating the effects of joule and viscous heating on the unsteady MHD Maxwell fluid flow via an angled stretched sheet. Their research concluded that magnetic factors contribute to a deceleration of the temperature profile. This insight is valuable in the design of electromagnetic systems where control over heat generation and dissipation is critical, such as in the cooling of high-performance computing devices or in magnetic braking systems. Finally, Ishak et al. [10] examined the effects of a heat source and sink, along with the chemical interactions of Maxwell fluid over a stretched porous cylinder. They determined that a viscoelastic fluid characteristic leads to a decreasing velocity profile. This finding is particularly relevant in chemical engineering processes where the interaction between fluid mechanics and thermal properties can significantly impact reaction rates and product quality.

In summary, the CC model and its associated research have provided critical insights into the behavior of Maxwell fluids under various conditions. These studies collectively demonstrate the model’s applicability across a range of disciplines, offering a more accurate understanding of heat transfer processes. This understanding is crucial for the development of advanced materials and technologies, particularly in fields that require precise thermal management and control. The continued exploration of the CC model promises to yield further advancements in our ability to design systems that effectively manage heat, ultimately leading to more efficient and reliable technologies.

Fluids that have nano-particles suspended in them are called nanofluids. Metallic and metal oxide particles, carbide particles, and even non-metallic entities such as carbon nanotubes are all considered nano-particles. It was Choi and Eastman [1] who initially reported onto these fluids. The goal here is to reduce particle concentration while maintaining good cooling (for stability). Heat exchangers in many different kinds of products (aircraft, home appliances, electronics, etc.) make use of nanofluids to the medical and technological industries, we have these fluids. Nanofluids are an endlessly fascinating area of study. Extensive studies have been conducted to modify the types of particles that are accessible, and various combinations of nano-particles have been utilised to enhance effectiveness. The goal of creating nanofluid composites is to improve the rheological or thermal conductivity of individual nano-particles [16]. Improving the thermal conductivity and rheological properties of a nanofluid is possible by constructing a composite nanofluid. Achieving this requires meticulously crafting an ideal blend of nano-particles [17].

The addition of thermally conductive nanoparticles to a nanofluid does not guarantee improved rheological characteristics. Therefore, the total capability, stability, and efficacy of the nanofluid can be improved by including nano-particles with varying rheological or thermal characteristics [18]. An example of a material with low thermal conductivity but high chemical inertness and stability is Al2O3. Meanwhile, particles with a higher thermal conductivity, instability, and chemical reactivity include aluminium, silver, copper, and others. Therefore, hybrid nanofluids are formed by mixing these nano-particles with various physical and chemical linkages. Nuclear safety, the pharmaceutical sector, electronic heater cooling, and many more fields find use for them.

2. Review of Literature

A major step forward in the introduction of hybrid and ternary hybrid nanofluids has been achieved in this work. Nanofluids are a typical liquid with several uses in fields as diverse as biotechnology, heat pumps, nuclear power plants, and heat exchangers. As demonstrated by Hayat and Nadeem [3], hybrid nanofluids outperform single-particle nanofluids when utilised as coolants. Furthermore, Chamka investigated the use of a hybrid nanofluid for convection heat transfer within the square cavity [5]. Research by Bahiraei and Mazaheri [2] examined the consequences of a hybrid nanofluid in an area where nano-particles of graphene were added to water. In contrast, tri-particle nanofluids of varying shapes were utilised by Arif et al. [4]. He found evidence of a 133 percent improvement in the heat transmission rate. Nano molecules with a diameter of two nanometers or less are considered hybrid nano molecules. Hybridity nanofluid is the name given to the liquid that is created by compound nanomaterials. In an effort to fix any mono nanofluid degradation, a novel nanofluid layer investigation employs a diversified structural addition.

When used for a variety of thermal applications, hybrid nanofluids effectively lower temperatures. One new area of nanotechnology is hybrid nanofluids, which are made by dissolving two distinct kinds of NPs in a basic liquid. Major study subjects that these pertain to include solar energy, HVAC applications, temperature change, heat pipes, electric cooling, generators, generators, refrigeration machinery and manufacturing, electric vehicles, nuclear cooling, transformer cooling, and electric cooling. Biomedical, space, and maritime systems all make use of hybrid nanofluids [20]. Alumina and copper were used to study the degree of weight and heat transmission in three-dimensional motion of nanofluids mixed on a revolving disc in a uniform magnetic area. drinks that contain nanoparticles in a water suspension. Previous research was used to organise and study several types of complex nanofluids [27]. Research shows that compared to base and mono nanofluids, hybrid nanofluids have superior heat transport and rheological properties [28].

Metallic substances can be stabilised, melted, pumped, and mixed by means of magnetism. In large-scale firm projection and refining operations, this offers non-contact metal advancement control [29]. The metallurgical field's magnetohydrodynamics (MHD) software has been transformed by the drive for more widespread application and the development of steel, aluminium, and super alloys with improved behaviour. Three classic bundles are detailed in this section. Their commercial significance and strong interest in fluid mechanics led to their selection [30]. Think about magnetic agitation first. To mix the ingot's partially solidified liquid sites, a magnetic disc that spins around the work surface is utilised. Made from a base liquid and nano-particles [31], the nanofluid that was invented utilising Choi and Eastman [1] is an artificial colloid. Nanoparticles, which are typically less than 100 nm, have a far better thermal conductivity than other fluids. The conventional generalisation of heat transmission in the simplest liquids is the basis for nanoparticle formation [32]. Oils, ointments, bioliquids, polymer solutions, water, natural refreshments (such as ethylene, diethylene glycol, refrigerants, etc.), and strange liquids for unexpected places are all examples of base fluids [33].

An innovative approach, when coupled with its utilisation, produces a synthetic substance that holds significant value in the field of biomedicine. Due to their optical characteristics, thermal taste of pictures, and unexpected interest in numerous possible image-related applications, tuned-shape hybrid nanodevices provide enormous control. With the resources available, NP can improve treatment outcomes by increasing tumour exposure to recovery entrepreneurs and extending break-in time. The organisation is an evaluation entrepreneur for diagnostics, optics, photoacoustic, and MRI imaging. One possible usage is in the delivery of drugs. To enhance tumour absorption and prevent the delivered drug from degradation, Xuan and Roetzel [11] utilised it. Per Kandasamy et al. [12], TiO_2 nano-particles and gold nano-particles can be used in the biomedical sciences, such as in photodynamic and ultrasound treatments for cancer.

Using alternating magnetic fields to heat the magnetic nanoparticles produced in tumours, Seo et al. [13] investigated their properties. Because human tissues are so little magnetically conductive, the effects of magnetic fields are negligible. But it's not out of the question that electromagnetic fields could induce eddy currents in any kind of living tissue. Additional studies on nanofluids' MHD peristaltic flow have also been conducted [34]-[35]. As we have shown, heat convection and magnetic flux are two important factors that affect double diffusion convection [36].

None of the fourth-grade fluids with double diffusion and inclined MHD have been investigated hence far. Based on the findings, this study potentially broadened the scope of prior work on existing fluid models. Flows of fluids of different densities are relevant in the medical sciences [37]. The motivation for this work came from a desire to better understand how the human body handles flows during related procedures like thallium stress testing and CT angiography. To begin, we checked for coronary artery blockages by injecting a dye and taking an X-ray of the arteries [38]. After that, researchers put a radioactive liquid (a radioisotope) into a human vein to measure the heart's blood flow during rest and exercise. The fields of biology, physics, and nanoscience (the study of materials) were all interwoven in this investigation [39].

The temperature profiles, solute concentrations, and nano-particle fractions were examined using figures. A important source, thermophoresis, was created when the temperature of hot gas was different from that of cold surfaces [40]. Particles were also propelled toward the cool surfaces as a result of this. Notably, the heat transmission in this study varied with the thermophoresis Nt parameter. As seen in Figure 5, the temperature profiles grew as Nt values rose. Figure 5 shows the same kind of impacts when looking at the Dufour NTC parameter. The heat flow that occurs anytime a concentration gradient is applied to a chemical system is physically described by the Dufour effect, which is also called the diffusion thermo effect [41]. Figure 6 shows that as Nb and NCT values increased, the solute concentration profiles shrank. This action was taken because of the close relationship between Nb and NCT. In addition, it happened because the solid nano-particles were scattered and their concentration was diminished as a result of random motions interacting with them through micro-mixing and random collisions. To observe the impacts of the nano-particle fractions, Figure 7 was made [42].

Fiber extrusion, paper technology, polymer film production, wire, and the plastics industry are just a few of the various operations that make use of stretching film. Changing the rate of cooling allows sheets of varying thicknesses to be stretched. In order to get high-quality sheets, the cooling rate is crucial [43]. The key concept lies in the introduction of better electric-powered conductivity of nano-particles compared to bottom fluid. A few high-heat conducting nano-particles are silver (Ag), Copper (Cu), Magnesium oxide (MgO), Titanium dioxide (TiO_2), CNTs, cobalt ferrite ($CoFe_2O_4$), and many other High Thermal Conductivity Nano-particles [14].

In the industrial sector, titanium dioxide is ideal as the lightest material; it is non-toxic and matches usage. Hybrid nanofluid used frequently contains MgO. Babu et al. [22] studied $CoFe_2O_4$ and Fe_3O_4 inside a water and ethyl glycol base channel. Sahu et al. [23] analyzed the cooling process of condensed CNT, Al_2O_2, and graphene as nano-particles. The experimental results were satisfactory when the tri-particle nanofluid with water as the base fluid was used in this work. Titanium dioxide, cobalt ferrite, and magnesium oxide make up the nanoparticles. In addition to the current nanofluid's modification capability, a more efficient coolant is still required [44]. A breakthrough in this area is hybrid nanofluid. The efficiency of the heat exchange process has been the subject of numerous experiments [15]. In order to achieve the required thickness, this paper employs a nanofluid flow to cool the stretching sheet. Typical industrial settings with magnetic fields are comparable. A crucial role is also played by the field.

3. Mathematical Formulation

Three particles, Al_2O_3, $CoFe_2O_4$, and TiO_2, as nano-particles and water considered base fluid, can pass through the elongated sheet. Nanofluid is assumed to be incompressible, and fluid flow can be considered laminar. The flow velocity along the stretching sheet is formulated as $U_\infty = ax$. An MHD of modulus B_0 is applied perpendicularly to the direction of the sheet. Applying Prandtl boundary layer equations, we get (Figure 1):

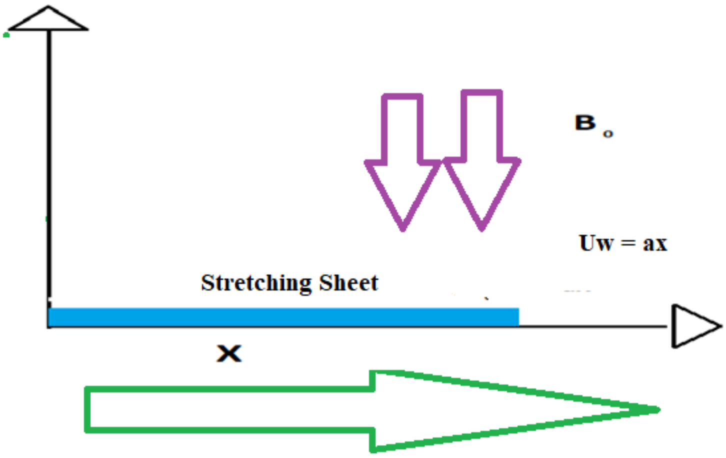

Figure 1: Flow of stretching sheet along ternary nanofluid

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\ \frac{\partial u}{\partial y}=\ \vartheta_{tnf}\frac{\partial^2 u}{\partial y^2}+\ \frac{\sigma_{tnf}{B_0}^2u}{\rho_{tnf}} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\ \frac{\partial T}{\partial y}=\ k_{nf}\frac{\partial^2 T}{\partial y^2}\ +Q_C(T-T_\infty) \tag{3}$$

Along with boundary conditions:

At y = 0, u=U$_\infty$, v = 0, $\frac{\partial T}{\partial y}=-h_t(T_w-T)$ (4)
And y→∞, u→ $0, T\rightarrow T_\infty$

$$Cf_x=\frac{\mu_{tnf}}{\mu_f(ax)^2}\left[\frac{\partial u}{\partial y}\right]_{y=0} \tag{5}$$

$$Nu_x=-x\frac{k_{tnf}}{k_f(T_w-T_\infty)}\left[\frac{\partial T}{\partial y}\right]_{y=0} \tag{6}$$

$$\rho_{nf}=\ \{(1-\emptyset_1)\{(1-\emptyset_2)\left[(1-\emptyset_3)\rho_f+\emptyset_3\ \rho_3\right]+\ \emptyset_2\rho_2\}+\ \emptyset_1\rho_1 \tag{7}$$

$$\mu=\frac{\mu_f}{(1-\emptyset_1)^{2.5}\ \ (1-\emptyset_2)^{2.5}\ (1-\emptyset_3)^{2.5}} \tag{8}$$

$$\frac{k_{tnf}}{K_{hnf}}=\frac{(k_1+2k_{hnf})-2\emptyset_1\ (k_{hnf}-k_1)}{(k_1+2k_{hnf})+\ \emptyset_1\ (k_{hnf}-k_1)} \tag{9}$$

$$\frac{k_{hnf}}{K_{nf}}=\frac{(k_2+2k_{nf})-2\emptyset_2\ (k_{nf}-k_2)}{(k_2+2k_{nf})+\ \emptyset_2\ (k_{nf}-k_2)} \tag{10}$$

$$\frac{k_{nf}}{K_f}=\frac{(k_3+2k_f)-2\emptyset_3\ (k_f-k_3)}{(k_3+2k_f)+\ \emptyset_3\ (k_f-k_3)} \tag{11}$$

Electrical conductivity$\frac{\sigma_{tnf}}{\sigma_{nf}}=\ \frac{(1+2\emptyset_1)\sigma_1+(1-2\emptyset_1)\sigma_{hnf}}{(1-\emptyset_1)\sigma_1+(1+\emptyset_1)\sigma_{hnf}}$ (12)

$$\frac{\sigma_{hnf}}{\sigma_{nf}}=\ \frac{(1+2\emptyset_2)\sigma_2+(1-2\emptyset_2)\sigma_{nf}}{(1-\emptyset_2)\sigma_2+(1+\emptyset_2)\sigma_{nf}} \tag{13}$$

$$\frac{\sigma_{nf}}{\sigma_f}=\ \frac{(1+2\emptyset_3)\sigma_3+(1-2\emptyset_3)\sigma_f}{(1-\emptyset_3)\sigma_3+(1+\emptyset_3)\sigma_f} \tag{14}$$

3.1. Conversion of equations

Dimensionless variables are introduced to solve the given partial differential equations.
$\psi=(a\ \vartheta_{tnf})^{1/2}xf(\eta)$

$\eta=(\frac{a}{\vartheta_{tnf}})^{1/2}y$

$u=ax\ f'(\eta)$

$v=\ -\left(a\vartheta_{tnf}\right)^{\frac{1}{2}}f\ (\eta)$

where ψ(x, y) is stream function and $u=\frac{\partial\psi}{\partial y}$ and $v=\ -\frac{\partial\psi}{\partial x}$, η is the similarity variable, and $\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$

$$\frac{\mu_{tnf}}{\mu_f}f'''+\frac{\rho_{tnf}}{\rho_f}(ff''-f'^2)-\frac{\sigma_{tnf}}{\sigma_f}Mf'=0 \tag{15}$$

$$\frac{k_{tnf}}{k_f}\theta'' + Prf\theta' + Q\theta = 0 \qquad (16)$$

$$f'(1) = 1,\ f(0) = 0,\ \theta'(0) = -\gamma\frac{k_{tnf}}{k_f}\big(1-\theta(0)\big),\ f'(\infty) = 0,\ \theta(\infty) = 0 \qquad (17)$$

$$C_f Re_x^{1/2} = \frac{\mu_{tnf}}{\mu_f} f''(0). \qquad Nu Re_x^{-1/2} = -\frac{k_{tnf}}{K_f}\theta'(0)$$

and

$$(Rex)_f = \frac{U_w x}{\vartheta_f}\ ;\ Q = \frac{Q_c}{T_w - T_\infty}\ ; Pr = \frac{\vartheta_f}{\alpha_f}\ ;\ M = \frac{\sigma B_0^2}{2U\rho_f}\ ;$$

3.2. Solution of the equation

Runge-Kutta method in fourth order accompanied by shooting technique was employed to solve equations (15) and (16), together with boundary conditions. Maleque and Sattar [24] provided a similar solution for porous plates. They initiated a series of guesses for f’(0). At each iteration loop, η is obtained from ηn+1 =η n+Δη.

4. Results and Discussions

By applying the RKF approach, we are able to resolve the ordinary differential equations (15). The new findings are highly compatible with the old ones. At each point inside this zone, the flow characteristics are drastically altered as they near the surface, resulting in a wall thickness reduction proportional to the velocity. Another parameter that reduces the temperature gradient is the wall thickness. This is because, in comparison to thinner sections, thicker ones transfer less heat energy to the fluid flow.

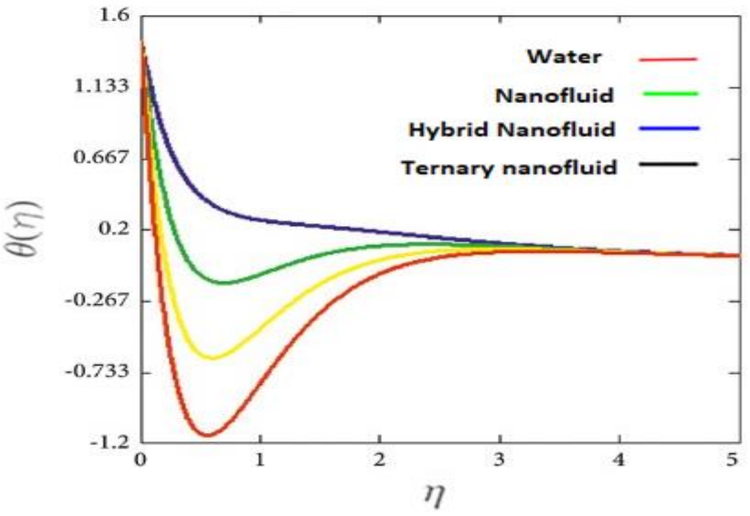

Figure 2: Base fluid, hybrid, ternary, and nanofluid comparison

In Figures 3 and 5, we can see the ternary nanofluid's profile of changed velocity and temperature. Upon initialization, the Lorentz force encounters flow. The force is proportional to the magnitude of the MHD. When fluids are in motion, it acts as a resistance force. The result is a flattening of the velocity profile. Deceleration allows cooling to occur over an extended period of time. Along with this, the heat transmission coefficient increases. An increase in the magnetic field was discovered to decrease the mobility of nanoparticles. An electrically conductive nano-particle experiences an increase in the Lorentz force a form of resistive force when it encounters a vertical magnetic field. One way to control the speed of a fluid is to take advantage of its magnetic feature. The result is that the fluid velocity can decrease with an increase to M. Similarly, as M increases, the velocity gradient decreases, a Lorentz force forms, and the boundary-layer thickness and heat conduction process are amplified, resulting in an increase in thermal dispersion [45].

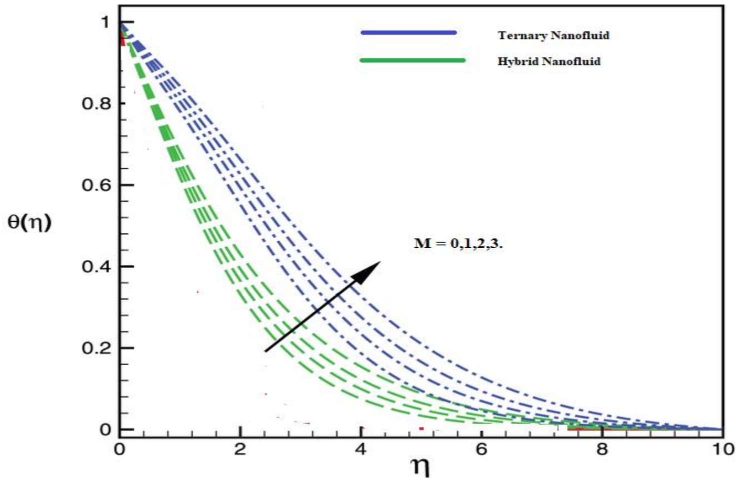

Figure 3: Variation in temperature with magnetic field

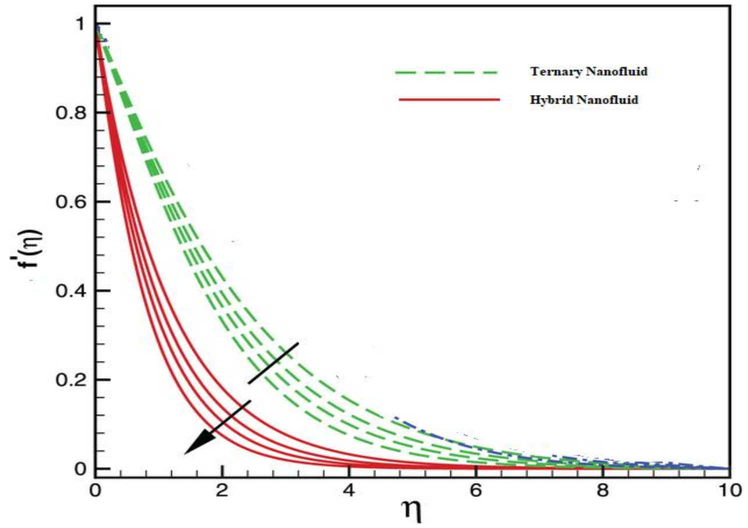

Figure 4: Magnetic field-induced velocity variation

Results showing how the concentration of nanoparticles affects dimensionless velocity and temperature are shown in Figures 5 and 6, respectively. The base fluid concentration of nano-particles grows in relation to their volume, which in turn enhances the fluid's heat-carrying capacity. A better option for cooling equipment is the result of this. Its photocatalytic characteristics make it an effective cooling when combined with other nano-particles, but it is also utilised alone. It provides excellent potential for stability and is inert as well. The density of nanofluid drops as it gets hot, which speeds up its flow rate. The flow rate is thus increased as the concentration of nanoparticles is raised because this results in a steeper velocity profile. Trapping is an extremely unusual occurrence in peristaltic propellant flows. The process begins with the formation of a fluid mass that is internally moving and surrounded by streamlines caused by peristaltic waves.

A large amount of fluid and property can be captured by utilising peristaltic waves that have high flow rates and large obstructions. Here we can see how fluid, nanofluid, hybrid, and tri-hybrid nanofluids compare in terms of heat transfer performance. The tri-hybrid nanofluid outperforms the other in terms of heat transmission because it makes use of various nano-particles linked by diverse chemical bonds, each of which contributes to the enhancement of heat transfer in its own unique way. Since TiO2 is both highly thermally conductive and photocatalytic in this mixture, it is primarily responsible for the increased heat conduction. Incorporating SiO2 improves TiO2's catalytic activity, allowing it to transmit more heat, while Al O2 3 ensures the fluid's chemical inertness and stability.

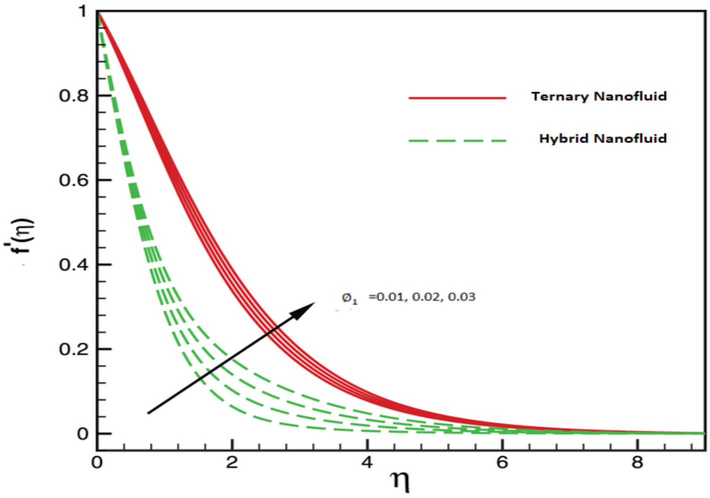

Figure 5: Variation in velocity with nano-particle concentration

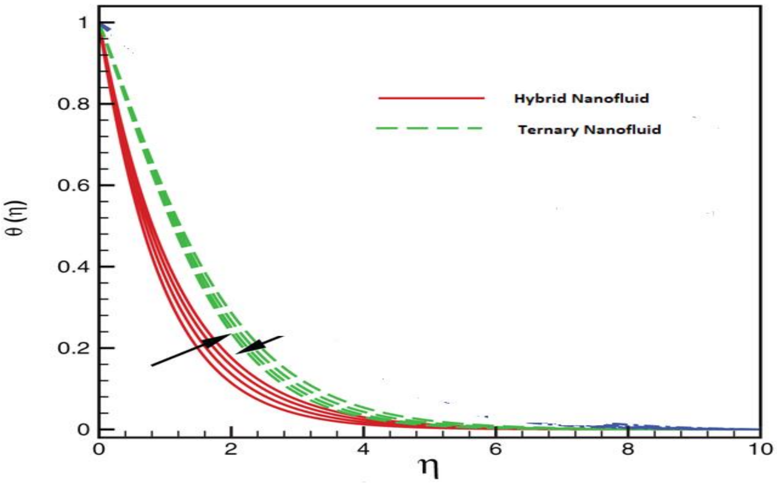

Figure 6: Variation in temperature with nano-particle concentration

Figures 7 and 8 show the variation in the skin friction coefficient and Nusselt number values due to changes in the magnetic field. The Lorentz force, which serves as a frictional force, is generated by MHD. These forces, which boost the kinetic energy of flowing fluids, are produced by an increase in the magnetic field. Consequently, this force causes an increase in the skin friction coefficient. When the magnetic field strength is increased, the skin friction coefficient also increases significantly. This result occurs because heat transfer through conduction decreases with increasing nano-particle concentration, φ_1, as seen in Figure 2 and Figure 6, showing that the value of the Nusselt number increases with an increase in MHD and φ_1. This happens when the Lorentz force, which opposes the flow of fluids and their intensities in the velocity profile, is amplified by increasing values of the magnetic parameter.

The figure clearly demonstrates that when the value of a magnetic parameter grows, its effect on the distributions of temperature and concentration becomes more pronounced. This ensures the appliance stays at the ideal temperature for its whole lifespan and improves heat absorption. The photocatalytic properties of TiO2 make it an excellent heat conductor; as a result, nanofluids containing TiO2 are increasingly being utilised as coolants. Additionally, the nanofluid is more stable due to its chemical inertness. Reducing air pollution can be achieved by employing this combination as a coolant in motor vehicles, as previously mentioned. Reduced in density and increasing their heat conductivity, these nano-particles float effortlessly in the nanofluid. As these less dense nano-particles drag the fluid in the same direction as their own motion, the fluid's flow velocity increases.

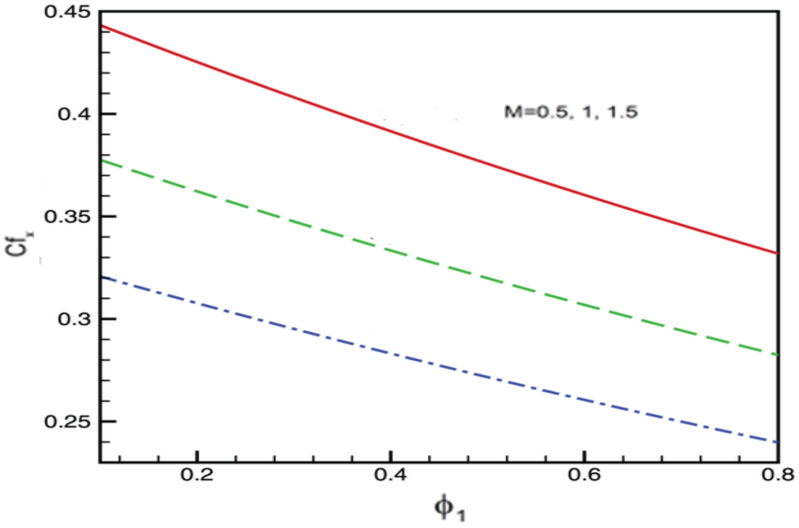

Figure 7: Nanoparticle concentration and magnetic field affect skin friction coefficient

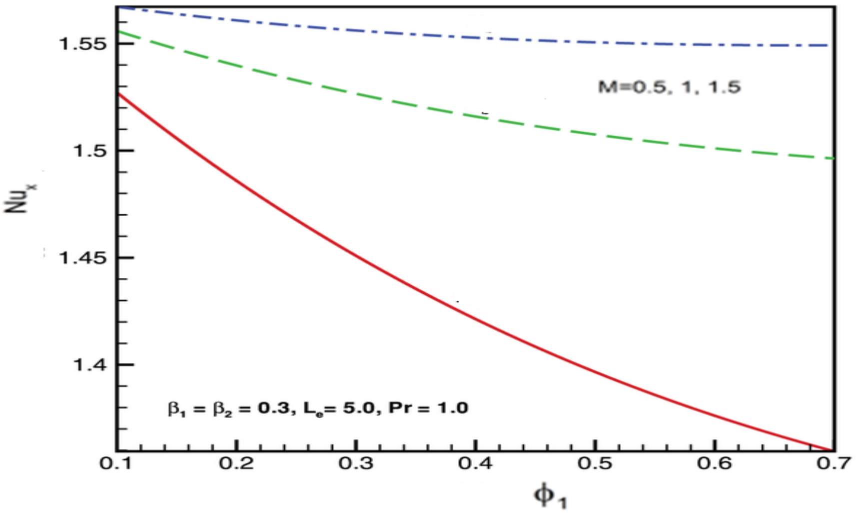

Figure 8: Change in Nusselt number with magnetic field and nanoparticle concentration

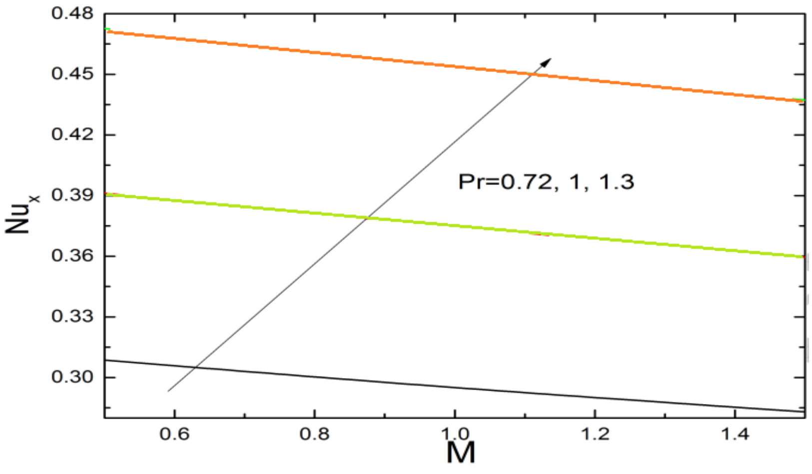

Figure 9: Magnetic field and Prandtl number affect Nusselt number.

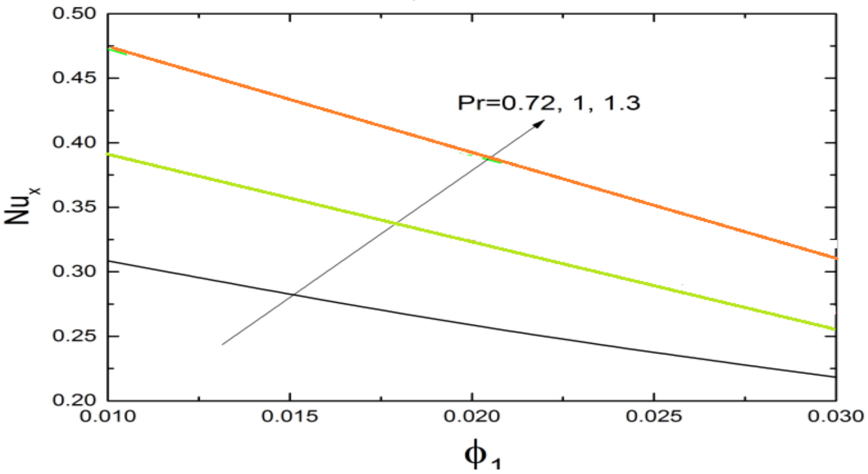

Figure 10: Magnetic field and Prandtl number affect skin friction coefficient.

Both Figure 8 and Figure 10 demonstrate that when the Prandtl number increases, so does the Nusselt number. A similar small drop in the temperature distribution is seen when the mixed convection parameter is increased. Effects of the dimensionless fluid parameter on the distribution of velocities. As seen in Figure, the streamlines for the different values of Nb reveal that the trapped masses grew in both quantity and size as Nb got better, but they shrank as m got better (Figure 9). As seen in Figure 10, the various waveforms' streamlines are displayed. This work is compared to others in the existing literature.

5. Conclusion

The gist of the research highlights the superior performance of triparticle nanofluids as coolants, establishing their efficacy over traditional fluids. Incorporating nano-particles into the base fluid significantly enhances thermal properties, with an increase in nano-particle concentration directly correlating with improved heat transfer capabilities. This phenomenon can be attributed to the dispersed nano-particles' enhanced thermal conductivity and surface area, facilitating more efficient heat dissipation. The analysis of heat transfer characteristics and flow behaviour for TiO_2-SiO_2-Al_2O_3-H_2O tri-hybrid nanofluid past a linearly stretching sheet is conducted. It is possible to translate the controlling equations into ordinary differential equations by applying appropriate similarity transformations. Graphs are used for result interpretation after the RKF 45 method is used to solve the ensuing system of differential equations. By taking into account the suspension of several classes of nano-particles that are applicable to the real-world scenario, this work can be expanded to investigate the behaviour of non-Newtonian fluids in varied physical contexts.

Moreover, the influence of magnetic fields on these nanofluids introduces an additional dimension of control over both fluid velocity and thermal conductivity. The ability to manipulate these parameters through magnetic fields opens up new avenues for optimizing cooling systems, particularly in applications where precise thermal management is crucial. It is found that the magnetic field has a direct effect on velocity and heat conduction, and hence, a reduction in velocity with the increase in the magnitude of the magnetic field results in an increase in the value of the Nusselt number. Heat exchange in a hybrid nanofluid can be exchanged more quickly than a single nanofluid particle. Tri-hybrid nano-particles are the best coolant among all fluids. Furthermore, the study suggests that exploring different combinations of nano-particles could yield even better thermal performance. By experimenting with various types and ratios of nano-particles, researchers can tailor the thermal properties of the nanofluid to specific requirements, potentially leading to breakthroughs in cooling technology.

In summary, triparticle nanofluids present a promising advancement in thermal management. Their enhanced heat transfer properties and the tunability of magnetic fields and nano-particle combinations pave the way for innovative cooling solutions across various applications. Continued exploration and experimentation in this area will likely yield significant improvements in thermal efficiency, contributing to developing more effective and reliable cooling systems.

5.1. List of symbols

B_0	Magnetic field coefficient
$B(x)$	Variable magnetic field
σ	Electrical conductivity

f	Sub-script for flow
f'	Dimensionless velocity
f''	Shear stress
M	Magnetic field parameter
Pr	Prandtl number
Re	Reynolds number
Nu	Nusselt number
Q	Heat source/sink parameter
u	Velocity component along the x-axis.
v	Velocity component along the y-axis
u_0	Uniform velocity
$U(x)$	Stream velocity
ΔT	Change in temperature
nf	Nanofluid
hnf	Hybrid nanofluid
tnf	Ternary nanofluid
α	Thermal diffusibility
β	Thermal expansion coefficient
μ	Dynamic viscosity
n	Kinematic viscosity
θ	Dimensionless temperature
Ψ	Stream function
η	Similarity variable
f	Volume fraction of nano-particles

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